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Reply by Author to F. M. White*

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THE author appreciates the interest of F. M. White in his paper. It is unfortunate that the reference to Spalding's earlier work has been overlooked. Spalding's formula represents the constant shear version of the more generalized form derived in Ref. 1. It should be noted that, although this formula appears to correlate adequately with the velocity distribution in the whole pipe, it yields an incorrect shear distribution in the outer regions when it is applied consistently with the eddy viscosity proposed by Spalding. The seemingly adequate representation of velocities is due to the fact that up to 70 or 80% of the velocity distribution does lie in a nearly constant shear region, whereas at the tail of the distribution almost any smooth monotonic function will close the gap reasonably well. No attempt has been made in the paper to adjust the numerical constants to fit experimental data; rather, they were taken from the literature, i.e., $k_1 = 0.4$ and $k_2' = 5.1$, which in turn yields $k_2 = 7.7$.

It has been stated in the report and may be reiterated here that, even with the correction for the shear distribution, the application of the derived law must remain within the wall region. In regions where the convective terms become substantially larger than the leading term in the expression for the shear distribution the flow no longer depends on local conditions only and, therefore, it is outside the range of applicability of the proposed law.

The generalized form of the law of the wall,[1] when applied to such diversified cases as the injection problem and the investigation of the growth of the viscous sublayer, predicts results that are not refuted by experiments; this fact provides sufficient reason to believe that the results are not fortuitous as contended by the commentator.

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Received October 19, 1967.

* The Technical Comment by F. M. White, which was published in Vol. 6, No. 4, p. 767, April 1968, should have appeared with this Reply.

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Comment on "Pressure Distributions on Sharp Cones in Rarefied Hypersonic Flow"

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IN a recent Note, McCroskey[1] has given an interesting correlation of a large quantity of surface pressure data from various sources taken on sharp cones at hypersonic semi-rarefied conditions. McCroskey also compares the data with the weak interaction theory of Burke and Dowling[2] and the strong interaction theory of Stewartson.[3] In the region where it is valid, the weak interaction theory agrees reasonably with the data. The strong interaction theory, however, agrees poorly, particularly in the region where one would expect it to agree best. The purpose of the present Comment is to show that this disagreement is primarily due to an algebraic mistake in the analysis of the heat-transfer case in Ref. 3. This error was noted and corrected by the present author in Ref. 4.

The mistake in Ref. 3 occurs in passing from Eq. (4.21)† to Eqs. (4.23) and (4.24) [although there is apparently a misprint in Eq. (4.21)]. Equation (4.23) should read

$$[4/(1 + S_w)]\epsilon_0^2 = C$$

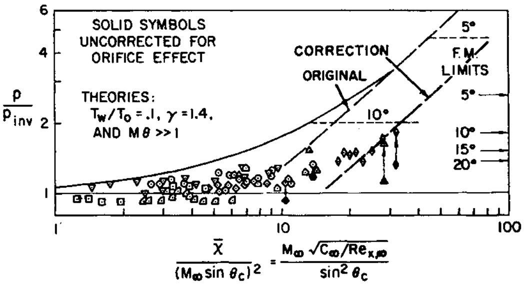

Fig. 1 Surface pressure distributions on highly cooled sharp cones.

Received November 29, 1967.

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† Numbered equations referred to herein are equations of Ref. 3; the notation is also that of Ref. 3.

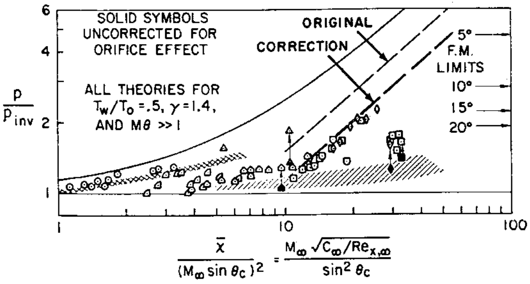

Fig. 2 Surface pressure distributions on sharp cones; $0.3 \lesssim T_w/T_0 \lesssim 1$.

and the factor (1 + S_w should be replaced by $8/(1 + S_w)$ in Eq. (4.24). It then follows that the factor $(2 + 2S_w)/(1 + 3S_w)$ appearing in Eqs. (4.25) and (4.26) should be replaced by $8/[1 + 3S_w)(1 + S_w)]$ and the right side of Eq. (4.27) should be multiplied by S_w.

Note that all the final results of Ref. 3 are incorrect for $S_w \neq 1$. The errors are most severe when the wall is highly cooled (i.e., $S_w \simeq 0$). In particular, the surface pressure p_c should be

$$p_c = [(1 + S_w)/2](p_c)_{\text{Ref. 3}}$$

In Figs. 1 and 2, the corrected surface pressure is superimposed on McCroskey's original figures. It appears that in both the highly cooled wall and the moderately cooled wall cases, there are regions in which the surface pressure from strong interaction theory agrees with the experimental data.

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Comment on "Carbon-Silica Reaction in Silica-Phenolic Composites"

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REFERENCE 1 considers the effect of subsurface reactions between silica and carbon upon the ablative response of silica-phenolic char layers. The analysis includes consideration of the energetic effect of silica-carbon reactions relative to other energy events prevailing in the immediate vicinity of the silica-carbon reaction zone. One of the conclusions reached in the reference from equilibrium chemistry considerations, that "the silica-carbon reaction is the dominant heat effect in the eroding wall," is believed misleading because it follows from considering only part of the over-all reaction events of importance. The present Comment gives further consideration to silica-carbon reaction events within the constraints of chemical equilibrium, and concludes that the net energetic effect is small and may be either endothermic or exothermic depending upon environmental gas composition. An opinion is also offered relative to the role played by silica-carbon reactions in affecting ablative performance of materials such as silica-phenolic.

In the analysis of Ref. 1, the endothermic silica-carbon reaction (reaction 1) is considered, but other (exothermic) reactions, such as reactions 2–4, between the products of reaction 1 and oxidizing species present for most ablative material applications are not considered. The oxidizing species represented in reactions 2–4 are typical of those present in boundary-layer flows for several applications of interest, i.e., earth entry or exhaust products in many rocket environments of current interest. Chemical equilibrium considerations dictate that all reactions in Table 1 (reactions 1–7) proceed strongly to the right for surface temperatures in the vicinity of 2,000°K and above. Hence, it is concluded that virtually all SiO generated below the surface will be reoxidized at the surface to form SiO_2 (l), provided the following conditions are satisfied: 1) that chemical equilibrium is achieved at the surface, and 2) that the rate of transfer of oxidizing species to the surface is at least as great as the rate of injection of SiO from below the surface. It is pertinent to consider the validity of these two provisions.

If chemical kinetic effects are such as to cause reactions 2–4 to proceed more slowly than reaction 1, the conclusion of Ref. 1 must be accepted to the extent that kinetic mechanisms dominate. We are not aware of evidence that reactions 2–4 are slower than reaction 1, and since they occur at the surface where temperature levels are generally higher than in depth, it seems reasonable to expect reactions 2–4 to be at least as fast as reaction 1. The second provision to be considered relates to the quantity of oxidizing species available and this clearly depends upon the environment. Our experience with analyzing silica-phenolic response in air, and rocket nozzle environments for oxygen-hydrogen and nitrogen tetroxide-Aerozine propellants,[2,3] indicates that sufficient oxygen is available to reoxidize virtually all SiO at the surface for a wide range of pressure and heat flux levels. Consideration of advanced propellant systems such as oxygen difluoride-diborane, however, does result in less SiO_2 formation at the ablating surface because less oxygen is available.

If virtually all SiO gas generated by reaction 1 is reoxidized at the surface by reactions such as 2, 3, or 4, then it is apparent that the net energy effect of silica-carbon reactions is more typically indicated by reactions 5, 6, or 7, and is small relative to the energy effect of reaction 1. This is the case because the net effect of the reactions is the oxidization of the carbonaceous char, with SiO_2 entering only as an intermediary. This conclusion has been analytically verified in computer experiments[3] we have performed with an ablation code which does account for the kinetically controlled silica-carbon reaction 1 in depth. Dual runs respectively allowing and excluding reaction 1 in depth resulted in substantially the same predicted surface temperature (presuming chemical equilibrium at the surface).

Table 1 Reactions of interest

Reaction[a]	Energy absorbed (+) or released (−) at 2200°K, kcal/mole	Reaction no.
$SiO_2(l) + C(s) \rightarrow SiO + CO$	+154	1
$SiO + \frac{1}{2}O_2 \rightarrow SiO_2(l)$	−182	2
$SiO + H_2O \rightarrow SiO_2(l) + H_2$	−122	3
$SiO + CO_2 \rightarrow SiO_2(l) + CO$	−116	4
$C(s) + \frac{1}{2}O_2 \rightarrow CO$	−29	5
$C(s) + H_2O \rightarrow CO + H_2$	+32	6
$C(s) + CO_2 \rightarrow 2\,CO$	+37	7

[a] (l) and (s) refer to liquid and solid, respectively. Unmarked species are gases.

Received January 26, 1968. Work supported by NASA Headquarters and NASA Lewis Research Center under contracts NAS7-218 and NAS3-7945, respectively.

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